Multiattribute Decisionmaking Using a Fuzzy Heuristic Approach

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Abstract—**Multiattribute decisionmaking (DM) is treated as a special kind of structured human problem solving. Emphasis is placed on the use of the available knowledge about utilities, which is obtained by combining heuristics and traditional aggregation methods. In this way, the problem of partial utilities and their interdependence may be solved. A fuzzy approach to DM is described, incorporating linguistic variables, relations, and algorithms. It is summarized in a formal model and illustrated by an example.**

I. INTRODUCTION

THE DECISIONMAKING process (DMP) may be described as the selection of a particular alternative from a set of possibles, so as to best satisfy the aims or goals of the environment. The DMP is a complex process, because of the identification, obtaining, and processing of information. It seemed that the only possible way for people to cope with complex problems is through an organized structural approach where the main problem is divided into a number of smaller, less complex subproblems. Most multiattribute decisionmaking follows such a structured approach [2], [8], [10].

Decomposing the alternatives onto different dimensions, usually called goals, attributes, performance variables, criteria, etc., allows the separate attributes to be evaluated independently. Finally, the total utility of an alternative is usually obtained by some aggregation procedure. The aggregated utility is then used as a basis for the selection of a particular alternative.

The problem of multiattribute DMP can be roughly summarized in the following four categories:

1) how to obtain knowledge about "utility,"
2) how to present or describe the knowledge,
3) how to present the alternatives,
4) how to carry out the DMP when the utilities are known.

In this field, research in psychology, business, engineering, medicine, etc. has produced a variety of decisionmaking (DM) models [9], [12], [15], [16], [18], [19]. The obvious advantages of existing DM techniques is that they provide an organized approach. The main disadvantages of these approaches are

1) interdependence of performance variables or goal nonorthogonality,
2) complicated and usually inadequate final aggregation of the utilities, and
3) the necessity of treating the subject in inconvenient numerical or financial terms, which is unsuitable for group or social DM.

In this paper we would like to make some suggestions concerning the above-mentioned problems and disadvantages. We believe that DM techniques must be and can be improved and made more human-like by using some results of fuzzy set theory and a heuristic approach [4], [17], [20], [21]. At the same time, a higher degree of operationality can be achieved.

The following points can be argued.

1) The utility of a performance variable is dependent on the levels of each of the other performance variables, so that the final aggregate utility is a complex function of performance variables.

2) The aggregate utility function is, in general, more complex than can be practically obtained by the combination of partial utilities. Therefore, a heuristic approach is needed to define the function. For example, some crucial points of the function must be identified on a question and answer basis. In this way, human heuristics can be built in and presented. Thus more adequate knowledge and more information about utilities may be obtained.

3) The human DMP is a vague and imprecise process. Decisions are not made according to the absolute quantities of attribute received but according to a subjective estimation of the "worth" of the levels of performance variables. Fuzzy set theory seems appropriate for the presentation of the DMP, easing particularly the man-technique interface by, for example, using linguistic variables instead of numbers. This could be of particular benefit when dealing with group or social decisionmaking with an emphasis on intangible benefits.

Regarding the above three arguments, the following decisionmaking model is suggested.

II. DESCRIPTION OF THE MODEL

The description of the model is in three parts: (A) presentation of the alternatives, (B) knowledge about utilities, (C) DMP. The description tries to introduce gradually

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some of the concepts mentioned above, especially those of linguistic variables.

A. Presentation of Alternatives

Let us say that every alternative in a given DM environment can be described with n attributes or performance characteristics. A performance space can be defined as the Cartesian product of n performance sets, $D_i; i = 1, 2, \cdots, n$. A point in the space is defined by the corresponding n-tuple $(d_1, d_2, d_3, \cdots, d_n) \in \mathscr{D}$, where $d_i \in D_i$ and can be denoted by $d^{(n)}$. If an alternative can be precisely defined, i.e., measured, along every performance variable, it can be presented as a point in the space $\mathscr{D}$. Generally, values of the performance characteristics are vaguely or imprecisely defined so that if an alternative A is a fuzzy subspace of $\mathscr{D}$, it is presented as the Cartesian product of fuzzy sets on D_i:

$$A = \mathscr{F}(D_1) \times \mathscr{F}(D_2) \times \cdots \times \mathscr{F}(D_n)$$

where $\mathscr{F}(D_i)$ indicates a fuzzy subset of D_i. A is a fuzzy subset of $\mathscr{D}$. The membership function $\mu_A(d^{(n)})$ is taken as, in accordance with the usual definition of fuzzy composition,

$$\mu_A(d^{(n)}) = \min_{i=1,\cdots,n} (\mu_A(d_i)).$$

Each alternative may now be visualized as a fuzzy region of the n-dimensional space $\mathscr{D}$. So, every point in $\mathscr{D}$ can be given a grade of membership of alternative A. The set of all possible alternatives is denoted by $\mathscr{A}$. $\mathscr{A}$ is a set of fuzzy sets.

B. Knowledge about Utilities

The total or aggregate utility of a point in the performance space $\mathscr{D}$ can be represented as a single scalar value or as a fuzzy set of values. With the fuzzy representation, the grades of membership at a particular point express the compatibility between that point and each utility value. So the utility function is an n-dimensional function which is more or less subjectively defined and unique to each individual or group. Furthermore, we assume that the aggregate utility function is not clearly defined in one's mind, and this definition is prone to inconsistency and imprecision, although some of the inconsistency, once recognized, may be removed. This is important when group utility or a social utility, which is an amalgamation of the utility of different groups, is sought. Also, because it is subjective and imprecise, people may be best able to express their feelings linguistically [20]. For these two reasons, aggregate utility functions may be appropriately presented as a fuzzy hypersurface.

Let us say that we may write down a universe of discourse U within which to discuss utility. The elements of the universe of discourse may be chosen by the decisionmaker himself, subject to a few simple rules. An element of the set U is denoted by u.

For example, U could be

$$U = \{\text{high, medium high, medium, medium low, low}\}.$$

The vocabulary may be extended by the use of hedges, such as "very," "quite," "more or less," etc. According to [20], the utility can be interpreted as a linguistic variable whose values "high," "low," etc., are the labels of fuzzy subsets on utility, which may be, for example, the interval [0, 1]. A fuzzy subset defines the meaning of its label. Hedges refine the scale by constructing new fuzzy subsets, usually by operations on existing ones. So a vocabulary is established which provides a natural description of imprecise values of utility.

Knowledge about utilities is denoted as a fuzzy relation Φ between $\mathscr{D}$ and U. The fuzzy relation Φ from $\mathscr{D} = \{d^{(n)}\}$ to $U = \{u\}$ is a fuzzy set on the Cartesian product $\mathscr{D} \times U$, characterized by a membership function $\mu_\Phi\{d^{(n)}, u\}$, by which each pair is assigned a truth value in the interval [0, 1].

The heuristic information used in the everyday DMP provides crucial points of the fuzzy relation Φ. These points should be obtained practically, by a question and answer technique, relying on simple linguistic variables. However, it is supposed that Φ will be presented as a table giving the utility of various points from $\mathscr{D}$. Missing points may be obtained by a chosen interpolation and later checked by the decisionmaker and changed if necessary.

In general, different groups have different Φ. When the common social utility Φ^s must be constructed, this can be done by operating upon all the separate Φ. The simplest method would be to use intersection, with or without weighting of the separate groups. As the number of groups increases, this method may become impracticable, since the regions of overlap will tend to diminish. At this stage, it would be reasonable to attempt to negotiate a joint Φ^s, allowing the groups to state explicitly where they are willing to trade benefits with other groups.

C. Decisionmaking Process

Each alternative will need to be measured along each performance dimension. The measurement will probably consist of a fuzzy subset of each performance set, so that a particular alternative A will be a fuzzy subset of $\mathscr{D}$, as explained in Section II-A.

If we use knowledge about utilities as a fuzzy relation Φ, from a given alternative $A \in \mathscr{A}$, a fuzzy subset v of U is inferred, presenting the value of alternative A:

$$v = \mathscr{A} \circ \Phi$$

or

$$\mu_v(u) = \max \{\min \{\mu_\Phi(d^{(n)}, u), \mu_A(u)\}\}.$$

The fuzzy set v presents the image of A in the standards or values presented by Φ, especially if Φ is Φ^s, an amalgamation of the difference pertaining to the individuals or groups involved in the DMP.

So, every alternative A gets more than a single value of utility within this fuzzy set v. Different utilities have different degrees of membership. To rank the alternatives, it is necessary to establish orderings among the sets to say "better," "equal," and "less good" among the fuzzy sets v. It is also desirable to say how much better one fuzzy set is than another. Sometimes the utility with the highest degree of membership could be taken as representative in order to make comparison among alternatives.

To rank the alternatives, we state the utility as a sentence, made up from the elements of U, the universe of discourse. This sentence expresses v linguistically. Alternatively, v may be presented graphically, allowing visual comparison of the alternatives.

III. Discussion

In this section the described model will be discussed in the light of existing models, approaches, and terms.

A. Performance Variables, Goals, and Utilities

When the scope of the DM problem is defined and the interest groups involved in the DMP are established, the identification of goals begins. This is in effect a structural decomposition of the problem, where components amenable to measurement must be identified. These measurable components are usually called performance variables or components D_i of the decision space $\mathscr{D}$.

But what are goals? Traditionally, a goal or subgoal means a target value or values of the performance variable or variables. Generally, a detailed goal G_i is a fuzzy set on the performance variable D_i. The membership function μ_{G_i} denotes for every element d_i from D_i a grade of membership in G_i. When the membership value is 1, this denotes the most desirable levels, which may be interpreted as the target values.

Partial utilities usually express the value which performance variables or goals have in the eyes of the environment or interest groups. In general, partial independent utilities can be interpreted as goals, where the utility increases as the target value is approached, so that the membership value of 1 means the highest utility. If some interdependences between utilities or goals exist, this is taken into account by, for example, weighting of the goals with respect to their relative importance.

The identification of performance variables, partial or detailed goals, and partial utilities is a very important part of the structured decomposition of the whole DMP and offers a natural basis for real understanding of the DM situation by the different people involved in the problem [11], [14]. Each performance variable has its own unit of measure, and a particular alternative may be measured along each of these units.

In the past, it has always been a problem to find how best to combine the diverse units of measure. Cost-benefit analysis has tried to do so by converting the measurement of each dimension to the common unit of money. This approach has led to inadequacies in the measurement of environmental and health hazards, for example, and other subjectively valued dimensions. Other methods have introduced a common dimensionless scale of scoring, which combined tangible and intangible effects of an alternative, although still requiring that all dimensions should be measured on cardinal scales [12].

To resolve this, we take the fuzzy approach and define for each performance variable its own universe of discourse. In many cases this will be equivalent to a continuous numerical unit of measure. But in others, some variables can take only discrete values which would constitute the universe of discourse, and in other cases where variables can best be described linguistically, their universe of discourse will be a set of appropriate adjectives. Instead of translating "tangible" and "intangible" quantities into the same common unit, we allow them to remain in their own most suitable language.

B. Aggregation and Aggregate Utility

Often, to determine the value of a particular alternative, it would be measured along the performance variables, so that its partial utilities may be obtained. The partial utilities must be combined so as to demonstrate the alternative's ability to satisfy the overall goal. The usual procedure is by linear combination, although multiplicative, conjunctive, disjunctive, and a variety of other rules have appeared [12], [19]. The traditional approach is thus to determine an overall utility of an alternative by separating out performance dimensions, measuring partial utility along each, and then recombining these so as to give an overall picture [18].

Traditional aggregation techniques such as weighted addition require, firstly, that the utility of any performance variable be independent of changes in the levels of all other performance variables, and secondly, that the trade-off rate is constant under any circumstances. These strict requirements, which amount to goal orthogonality, must be satisfied for such an aggregation method to be valid. Unfortunately, these requirements are often taken for granted and may not be thoroughly tested, despite their being difficult to satisfy in practice.

Weighted addition is not the only method used, and a few others have been mentioned already. Other, more esoteric methods involve geometric, harmonic, and higher order means, again with weighting. But no psychological evidence exists to show that these are anything more than mathematical artifacts [15].

The approach suggested in this paper differs from the traditional one in some important respects. The fundamental postulate made is that

> Partial utilities cannot be correctly and operationally aggregated by general mathematical functions because the aggregation methods actually used are unique to each individual and too complex to be satisfactorily described by an arbitrary combination rule.

It is accepted that the environment must have known aims, and it desires to choose one alternative from many which will best fulfill those aims. The aims may be stated as goals and subgoals, and these may be broken down into measurable performance variables [11]. But, at this stage, where, traditionally, the partial utilities are measured to be aggregated later, a different approach is suggested. These performance variables define a decision space, and each point in the decision space has its own aggregate utility. Instead of decomposing the overall utility onto the separate performance variables, we propose to measure utility as a function of all the performance variables. This sounds like a daunting task, but the use of heuristics and simplifying assumptions allows us to make decisions in practice, and it is this human method which we seek.

We propose that the decomposition of goals yields only the relevant performance variables, and we do not seek targets along them. This is because of the complexity of the subjectively integrated aggregation functions, called Φ, which allow trade-offs between performance variables, so that targets cannot be uniquely defined.

This aggregation function Φ operates on all the relevant performance variables to produce an overall aggregate utility. Traditionally, Φ is in two parts:

1) operates separately on each performance variable yielding partial utilities, and
2) operates on partial utilities to yield aggregate utility.

We propose that Φ should consist of one part only, the purpose of which is to operate on performance variables yielding aggregate utility directly. This is also the main difference between our proposed approach and existing indifference methods [5] where the problem can be easily handled only in two dimensions.

The fuzzy relation Φ contains more information about utility than existing methods, because it contains within it the details of the aggregation method as well as utilities of separate dimensions. At best, partial utilities can supply only cross sections through Φ along the axes. By projecting onto $D_i \times U$, where D_i is any performance variable, we obtain a fuzzy subset of $D_i \times U$. This is roughly equivalent to the partial utility of "goal" of dimension D_i. But, by projecting Φ out onto the dimensions, we immediately lose all the information on aggregation. This is the dilemma of existing methods—the information on actual aggregation has been lost, and some arbitrary method must be substituted. We may break Φ down to produce partial utilities, but in doing so, we lose all the information on aggregation and so cannot recombine the partial utilities to produce a complete Φ. The relation Φ, which maps from $\mathscr{D}$ to U is irreversible, and the inverse mapping from U to $\mathscr{D}$ cannot be defined.

We have used the max-min composition rule, although this is not the only rule which could be constructed to map from the performance space onto the utility space. This rule has some undesirable features, as will become apparent during the discussion of the example. It does not interpolate adequately between the n-tuples and produces multiply peaked fuzzy sets as the final answers, instead of the value at the region between n-tuples where the alternative may be. This problem, though, requires further research and should not detract from the overall purpose of this work.

Since Φ contains all the information pertinent to the decision, we can see how constraints on the decision problem are built in. Constraints may be stated as levels of performance variables which are unacceptable or levels of two or more variables which, in combination, are also unacceptable. Such statements are really heuristics describing regions of $\mathscr{D}$ where the utility is zero or undefined. The constraints may be obtained from Φ by projecting it onto $\mathscr{D}$. All regions of $\mathscr{D}$ with zero grade of membership in this projection have no image in U, and thus no utility. These regions are, by definition, excluded by the constraints.

Some decisionmaking techniques take goals and constraints as similar types of statement [1], [3], [13]. Both place requirements on any alternative which must be more or less satisfied. While this interpretation is valid and consistent under those regimes, it does not apply under our approach. Goals lead to the selection of the relevant performance variables which define the problem's decision space. Those regions of the decision space where utility is not defined represent the constraints.

C. How Φ can be Obtained

Man's ability as an information processor is very limited—we cannot handle more than half a dozen items of information at once in our short term memory. So, when faced with a decision requiring the balancing of more than a few performance dimensions, we must have some means of reducing the problem to a workable size. To do this, we use simple heuristics or "rules of thumb," selecting and evaluating the more important criteria, until the important information is at hand and a decision can be made. It is this process that we want to use—the human combination of rules, be they weighted addition, lexicographic, random, or whatever.

This heuristic decisionmaking process should be obtainable under a question and answer system, involving dialogue between the decisionmaker and the analyst, whose role could be taken by a computer. This process will require the decisionmaker to think very carefully about his decision algorithm and to remove inconsistency and explain arbitrariness within it. In order to write down the heuristics and take account of the imprecision, fuzzy set theory is used to express the concepts simply and concisely.

At the beginning, Φ consists of only a few crucial points for which the utility value can be picked up from the heuristics through conversation. Between the points, utility is obtained by a suitable interpolation by, for example, weighted addition. At the next stage, the decisionmaker checks the calculated utilities and changes them where necessary. Afterwards, the interpolation follows again. The cycle could be a man-machine one, and may be performed as many times as necessary, until the Φ fits the user's opinion.

D. The Role of Fuzzy Set Theory in DMP

In the above discussions, fuzzy set theory was used as a suitable tool for describing the subject. The DMP has many vague imprecise concepts which cannot be defined clearly or uniquely [5], [6], [13]. In such cases, fuzzy set theory is a suitable language not only for formally modeling the process, but especially for handling a human-oriented structured approach on the operational level of the DMP.

Any decisionmaking technique which uses ordinary mathematics necessarily requires the weights, scores, utilities, etc., of the goals and alternatives to be expressed in the language of numbers. Many people are ill at ease with figures and uncomfortable with talk of statistics, averages, and absolute quantities. An answer presented as a number to four significant figures can produce a spurious impression of accuracy to those all too aware of the imprecision of the input information they gave. The information which people have in their heads and which they use with great success every day in making decisions is expressed in mathematically intractible natural language. To require them to translate into the foreign tongue of numbers is a flaw little recognized in existing methods.

Fortunately, fuzzy set theory is powerful enough to allow people to say how they take decisions in natural language, which can then be translated into mathematics, if need be, when it comes to be used as a tool. Nearly all the tedious calculations with fuzzy sets can be done by computer. One of the biggest problems which probably remains in decision-making is in establishing the vocabulary of linguistic values. The determining of the membership function μ could be done through group discussion, seeking agreement among the people involved in the decisionmaking process.

IV. Example

The following example will illustrate a practical application of the approach. It may be summarized under seven headings, as follows.

1) Define problem and identify different interest groups of people involved in DM.
2) Identify performance variables and establish universe of discourse for each performance variable.
3) Identify heuristics and ideals for the separate groups.
4) Construct the utility relation Φ for every group.
5) Distinguish alternatives and measure along the performance variables.
6) Calculate utilities of individual alternatives for every group.
7) Rank alternatives on the basis of calculated utilities.

In general, all the steps (except the calculation of step 6, for example) are participative, and the decisionmaker would play a central role in each of them.

A. Problem Definition and Identification of the Interest Groups

To illustrate the essence of the described approach, the following simple practical problem has been selected:

A family wants to buy an electronic calculator. The interest groups involved are the parents and the children.

B. Identification of Performance Variables and Establishment of Universe of Discourse for each Variable and Utility

The two groups express what they are going to use the calculator for, i.e., what they expect in terms of a mixture of performance and goals. The parents want a calculator which is cheap and capable of performing simple domestic calculations. The children want to be able to do scientific schoolwork on the calculator, so performance matters. Size matters to both groups, as the calculator should be easy to handle and small enough to fit into handbag, pocket, or schoolbag.

So the performance variables and corresponding universes of discourse could be

$D1$ nature of calculator,

$D1 =$ {scientific, scientific-programmable, business, business-programmable, four-function only};

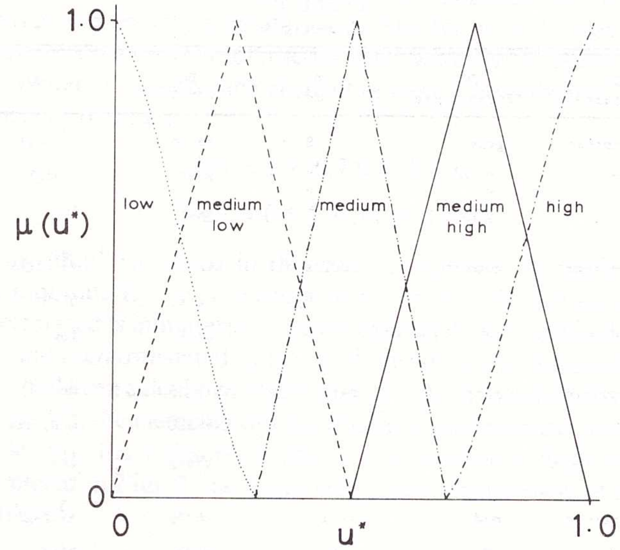

Fig. 1. Definitions of verbal utility values on U^*, [0, 1].

$D2$ performance which includes accuracy, computing speed, efficiency (how few key-strokes would be needed to solve a variety of calculations), etc.

$D2 =$ {good, medium, poor};

$D3$ size of calculator. Existing calculators would be grouped by size in the following five categories:

$D3 =$ {very small, small, medium small, medium large, large};

$D4$ price of calculator

$D4 =$ {cheap, reasonable, dear}.

The universe of discourse for utility is also derived naturally at this stage. For this example, the universe used is

$U =$ {high, med high, med, med low, low}

along with the hedges "and," "or," and "very," see [20]. Fig. 1 shows a graphical representation of the meaning of the linguistic values of the variable U.

C. Heuristics and Ideals for Each Group

This section follows smoothly from the previous one. The parents' heuristics for this example are as follows.

1) The nature of the calculator is not important.
2) If the calculator is cheap, its utility is at least medium, even if it is medium large and performs poorly.
3) If it is dear, it must be small and have good performance to be acceptable.
4) Large calculators are unacceptable.

The children's heuristics are as follows.

1) A scientific calculator is preferred.
2) Performance, so long as it is at least medium, is not important.
3) The size must not be large or small.
4) The price is not very important, but it must not be dear.

TABLE I
PARENTS

D1 Nature	D2 Performance	D3 Size	D4 Price	Utility
sc, non-sc	good	S	cheap	v^3 high
"	"	"	reas.	v high
"	"	"		
"	"	"	dear	low
sc, non-sc	good	med. small	cheap	v^2 high
"	"	"	reas.	high
"	"	"	dear	low
sc, non-sc	good	med. large	cheap	med.
"	"	"	reas.	med. low
"	"	"	cheap	low
sc, non-sc	med.	small	cheap	v^3 high
"	"	"	reas.	high
"	"	"	dear	v low
sc, non-sc	med.	med. small	cheap	high
"	"	"	reas.	med. high
"	"	"	dear	v low
sc, non-sc	med.	med. large	reas.	low
sc, non-sc	poor	small	reas.	med. high
sc, non-sc	good, med, poor	large	cheap, reas, dear	v^3 low

TABLE II
CHILDREN

D1 Nature	D2 Performance	D3 Size	D4 Price	Utility
sc	good	med. small med. large	cheap	v^3 high
"	"	"	reas.	v high
"	"	"	dear	med. high
sc	good	very small large	cheap	med. high
"	"	"	reas.	med.
"	"	"	dear	med.
sc	good	small	cheap	v high
"	"	"	reas.	high
"	"	"	dear	med.
sc	med.	med. small med. large	cheap	v high
"	"	"	reas.	high
"	"	"	dear	med.
sc	med.	very small large	reas.	med.
sc	med.	small	cheap	med. high
"	"	"	reas.	med. high
non.sc	good	med. small med. large	cheap	med. low
"	"	"	reas.	low
"	"	"	dear	v low
non-sc	good	small	cheap	low med.
"	"	"	reas.	low med.
"	"	"	dear	low
non-sc	med.	med. small med. large	cheap	low
"	"	"	reas.	v low
"	"	"	dear	v^3 low
non-sc	med.	small	cheap	v low
"	"	"	reas.	v^2 low
"	"	"	dear	v^3 low
sc, non-sc	poor	very small small med. small med. large large	cheap reas. dear	v^3 low

To establish the point of maximum utility as a reference position, both groups are asked to state their ideal calculator, whether or not it exists in practice. The parents' ideal is

> The ideal calculator would be small and cheap with good performance.

The children's ideal is

> The ideal calculator would be medium sized, scientific, and cheap. The performance should be good.

D. Construction of Utility Relation Φ for Every Group

From the universes of discourse in Section II, we can see that the performance space $\mathscr{D}$ will contain $5 \times 3 \times 5 \times 3 = 225$ n-tuples. To ask for the utility for each point separately would be an unreasonable task. But the heuristics in the previous section let us reduce the number of n-tuples. For example, for $D1$ only two values are important—scientific and nonscientific nature—instead of the original five values. Also, many points will have a near similar utility. For example, the children state that any calculator with poor performance is unacceptable, so immediately the number of n-tuples covering the region of "at least acceptable" utility is reduced to $2 \times 2 \times 5 \times 3 = 60$ points.

With the help of the heuristics, the utility of only a few crucial points needs to be determined. The utility of the other n-tuples in the space in between is determined by some chosen interpolation (usually weighted by an importance of the performance variables). This can be done interactively by computer. The most important calculated utilities are displayed to be checked by the groups and can be immediately changed if they do not match the groups' feelings. After that, a new cycle of interpolation, followed by a display of the new results, starts. When the groups agree with the utility (i.e., no changes are required), the utility relation Φ is ready to be applied to the evaluation of alternatives.

Part of Φ for the parents is shown in Table I and for their children in Table II. Note that "v high" means "very high," "v^2 high" means "very, very high," etc.

E. Description of Alternatives

Every alternative can be described as either an n-tuple in $\mathscr{D}$ or, if its measurement along the performance variable is vague, as a fuzzy subset on $\mathscr{D}$. Let us have three different types of calculators as alternatives, described as follows:

1) The first alternative is a calculator which has nonscientific nature, good performance, medium small size, and reasonable price. It can be presented as the following quadruple:

$$A1 = (\text{nonsc, good, med small, reas})$$

2) For a calculator chosen as the second alternative, performance is somewhere between medium and poor but

less poor than medium. Performance can be expressed as the fuzzy set $\mathscr{F}(D2) = \{0.6/\text{med}, 0.4/\text{poor}\}$. The values of other performance variables are nonfuzzy because it is easy to establish that the calculator is scientific, small, and reasonably priced. Using the Cartesian product operation, the calculator can be described by a fuzzy set containing two quadruples:

$$A2 = \{0.6/(\text{sc, med, small, reas}), 0.4/(\text{sc, poor, small, reas})\}$$

3) The third alternative can be described by fuzzy sets on all four performance variables:

$$\mathscr{F}(D_1) = \{0.6/\text{sc}, 0.3/\text{nonsc}\}$$

$$\mathscr{F}(D_2) = \{0.9/\text{good}, 0.2/\text{med}\}$$

$$\mathscr{F}(D_3) = \{0.5/\text{med small}, 0.5/\text{small}\}$$

$$\mathscr{F}(D_4) = \{0.3/\text{cheap}, 0.6/\text{reas}\}.$$

Their Cartesian product is

$$\begin{aligned} A3 = \{&0.3/(\text{sc, good, med small, cheap}), 0.5/(\text{sc, good, med small, reas}),\\ &0.3/(\text{sc, good, small, cheap}), 0.2/(\text{sc, good, small, reas}),\\ &0.2/(\text{sc, med, med small, cheap}), 0.2/(\text{sc, med, med small, reas}),\\ &0.2/(\text{sc, med, small, cheap}), 0.2/(\text{sc, med, small, reas}),\\ &0.3/(\text{nonsc, good, med small, cheap}), 0.3/(\text{nonsc, good, med small, reas}),\\ &0.3/(\text{nonsc, good, small, cheap}), 0.3/(\text{nonsc, good, small, reas}),\\ &0.2/(\text{nonsc, med, med small, cheap}), 0.2/(\text{nonsc, med, med small, reas}),\\ &0.2/(\text{nonsc, med, small, cheap}), 0.2/(\text{nonsc, med, small, reas})\}. \end{aligned}$$

F. Calculation of Utilities of Individual Alternatives for Every Group

The previous stages are intended to be interactive and participative, under the decisionmaker's control. Once the alternatives have been described as n-tuples or fuzzy sets, the subsequent calculations are relatively simple and can be efficiently done by computer.

Following the inference rule, for every alternative, its value v_{Ai} can be derived from the utility relation Φ^P (parents) or Φ^C (children). In general, v_{Ai} is a fuzzy set:

$A1$ parents $v^P_{A1} = \{\text{high}\}$

children $v^C_{A1} = \{\text{low}\}$

$A2$ parents $v^P_{A2} = \{0.6/\text{high}, 0.4/\text{med high}\}$

children $v^C_{A2} = \{0.6/\text{med high}, 0.4/v^3\ \text{low}\}$

$A3$ parents $v^P_{A3} = \{0.3/v^3\ \text{high}, 0.3/v^2\ \text{high}, 0.5/v\ \text{high}, 0.5/\text{high}, 0.2/\text{med high}\}$

children $v^C_{A3} = \{0.3/v^3\ \text{high}, 0.5/v\ \text{high}, 0.5/\text{high}, 0.2/\text{med high}, 0.3/\text{med low}, 0.3/\text{low}, 0.2/v\ \text{low}, 0.2/v^2\ \text{low}\}$.

G. Ranking Alternatives on the Basis of Calculated Utilities

The utilities of the alternatives are presented as the fuzzy sets v. The next, and final, step is to rank the fuzzy sets and so establish which is the best alternative.

To condense the v obtained above into something which approximates a sentence in natural language, we must introduce an intermediate stage. The terms in U may be defined on a universe of discourse U^* which is [0, 1] (see Fig. 1). We may express each element of v as a fuzzy subset of U^*, instead of U, e.g.,

$$\text{high} = 1/1, 0.7/0.9, 0.3/0.8$$

$$0.5/\text{high} = 0.5/1, 0.5/0.9, 0.3/0.8.$$

The "min" rule used in the example above is to preserve commutativity, i.e., the order in which the calculations are performed is immaterial. This means that if v is calculated on U and then translated to U^*, the same answer is obtained as if v had been calculated on U^* directly. Once translated from U to U^*, the elements of v may be combined using a "max" rule. For the v described above, this yields the fuzzy sets depicted in Figs. 2 and 3, allowing visual comparison of the utilities.

Thus the result is obtained that, for each alternative, the linguistic opinions are

$A1$ parents $v^P_{A1} =$ high

children $v^C_{A1} =$ low

$A2$ parents $v^P_{A2} =$ high or med high

children $v^C_{A2} =$ med high or very, very, very low

$A3$ parents $v^P_{A3} =$ high

children $v^C_{A3} =$ high or low but not very, very, very low.

The parents would seem to be required to choose between alternatives $A1$ and $A3$. By referring to the graphs, we see that the utility $A1$ is more truly high, since the truth of the statement that "the utility of $A3$ is high" is, by definition, less than unity.

We would expect the children to choose $A3$, since it offers the possibility of "high" utility, excluding "very, very, very low" utility. This impression is more or less confirmed by referring to Fig. 3. Alternative $A3$ may provide higher utility

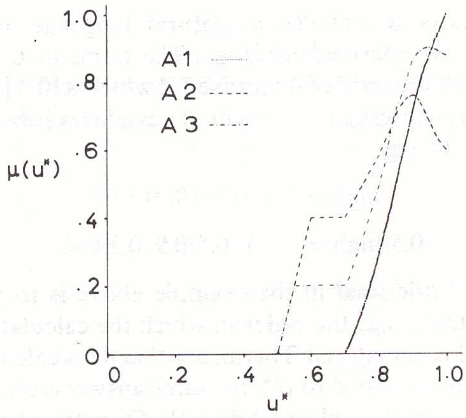

Fig. 2. Final utility for parents.

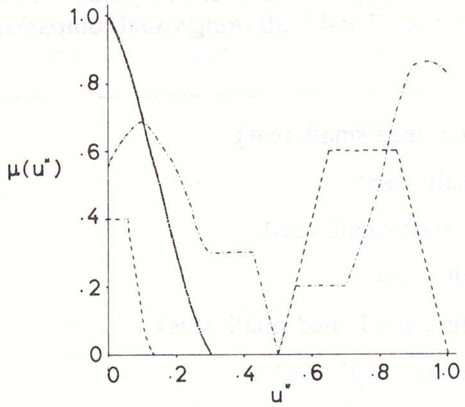

Fig. 3. Final utility for children.

than either of the other two, with less risk of exceptionally low utility. If we take the intersection of the fuzzy sets v as representing agreement among the interest groups, where intersection is defined as

$$\mu_{A \cap B}(u) = \min\,(\mu_A(u), \mu_B(u)),$$

then we find that alternative 3 is again best.

V. Conclusion

The decisionmaking approach outlined in this paper represents an attempt to move away from the traditional concept of partial utilities, based on numerical measurements. Fuzzy set theory has been applied to tackle two problems, 1) the interdependence of utility as expressed in heuristics, and 2) the retention of subjective measurements in natural language. The method is flexible enough to handle both fuzzy and nonfuzzy information and can indicate to the decisionmaker where he is not providing complete information.

A multiattribute decisionmaking problem has been presented to show how the approach works. At present, there are still difficulties in determining the correct values of Φ and in the final ranking of alternatives, but it is hoped that these problems may soon be surmounted.

The main aim of this fuzzy heuristic approach is to provide a decisionmaking technique which is more humane and may be operational as a special part of participative system analysis and design.

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